3-8 Videos Guide

3-8a

- The method of Lagrange multipliers
  - In $\mathbb{R}^2$ and $\mathbb{R}^3$, when optimizing a function f subject to the equation $g = c$, we use the fact that $\nabla f = \lambda \nabla g$ (λ is called the Lagrange multiplier)

Exercises:

- Find the maximum area for a rectangle inscribed in the ellipse $\frac{x^2}{3^2} + \frac{y^2}{4^2}$

3-8b

- Use Lagrange multipliers to find the extreme values of the function subject to the given constraint.
  - $f(x, y) = 3x + y; \quad x^2 + y^2 = 10$
  - $f(x, y) = xe^y; \quad x^2 + y^2 = 2$

3-8c

  - $f(x, y, z) = \ln(x^2 + 1) + \ln(y^2 + 1) + \ln(z^2 + 1); \quad x^2 + y^2 + z^2 = 12$

3-8d

- Use Lagrange multipliers to find the points on the cone $z^2 = x^2 + y^2$ that are closest to the point $(4, 2, 0)$.

3-8e

- When optimizing a function $f(x, y, z)$ subject to two constraints $g = k$ and $h = c$, we use $\nabla f = \lambda \nabla g + \mu \nabla h$

Exercise:

- Find the extreme values of f subject to both constraints.
  $f(x, y, z) = z; \quad x^2 + y^2 = z^2, \quad x + y + z = 24$